QUESTION OF MEASURING NONSTATIONARY THERMAL FLUXES

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A method is proposed for measuring time-varying thermal fluxes.

Methods are known for measuring nonstationary thermal fluxes that are based on analyzing the time dependence of the temperature in two and more sections of a plate or semi-bounded body [1-5]. One of the sections should be as close as possible to the sensor working surface, which will raise its sensitivity but makes difficult experimental realization. Heat propagation in a body is ordinarily assumed one-dimensional. In other cases the variable heat flux is determined by the mean bulk temperature of the body [5, 6] or by the temperature of the working surface [7]. Sometimes the method of a "dynamic thermocouple" is used to investigate heat transfer in a plasma jet, which permits heat flux measurement up to the thermocouple junction [6], which is the average quantity that can be utilized with a certain error to compute heating of the particles similar to the thermocouple junction in their properties [8].

Real sensors can correspond to the schemes considered only with definite errors.

It is most convenient to locate the thermocouple on the plate surface opposite to the working surface. A method, verified in model problems and in test, is examined below for such a sensor scheme.

Taking account of the temperature dependence of the material properties, the problem of plate heating by an arbitrarily varying thermal flux from the surface x = R while the surface x = 0 is heat insulated can be written as follows

$$c(t)\rho\frac{\partial t}{\partial\tau}=\frac{\partial}{\partial x}\left(\lambda(t)\frac{\partial t}{\partial x}\right), \tag{1}$$

$$c=c_0+c_1t,\quad \lambda=\lambda_0+\lambda_1t, \tag{2}$$

$$q|_{x=R}=q(\tau), \tag{3}$$

$$q|_{x=0}=0, \tag{4}$$

$$t|_{x=0}=f(t), \tag{5}$$

$$t|_{\tau=0}=t_0. \tag{6}$$

We will seek the solution of the problem in the form

$$t(x,\ \tau)=M+Qx+Nx^2+Gx^3+Sx^4, \tag{7}$$

where M, Q, N are functions of the time and G and S are constants.

Integrating (1) between 0 and R and utilizing (2), (3), and (7), we obtain

$$q(R,\ \tau)=\rho\left[M'\left(c_0R+c_1MR+c_1N\frac{R^3}{3}+c_1G\frac{R^4}{4}+\right.\right.$$
$$\left.\left.+c_1S\frac{R^5}{5}\right)+N'\left(c_0\frac{R^3}{3}+c_1M\frac{R^3}{3}+c_1N\frac{R^5}{5}+c_1G\frac{R^6}{6}+c_1S\frac{R^7}{7}\right)\right]. \tag{8}$$

Solving (1)-(6) by using (7), we find values of the coefficients in (7) and (8)

$$M=f(t), \tag{9}$$

$$Q=0, \tag{10}$$

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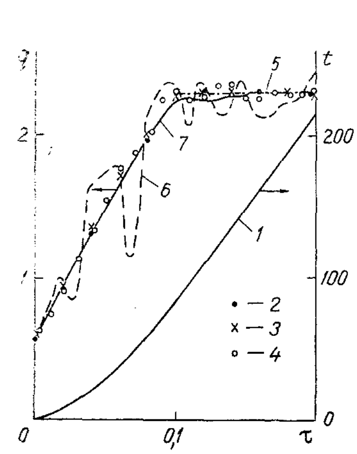

Fig. 1

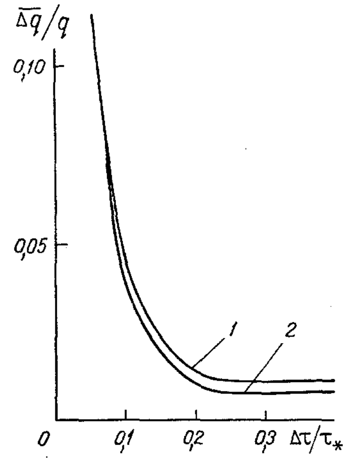

Fig. 2

Fig. 1. Time dependence of the temperature (x = 0) (1) and the heat flux (x = R) (2-7) (R = 5 mm): 2) Δτ = 0.04 sec, 3) 0.02, 4) 0.01, 6, 7) 0.002 sec, 6) time step for finding the derivative dt/dτ 0.002 sec, 7) 0.02; 5) exponential method. q, kW/cm², t, °C, τ, sec.

Fig. 2. Dependence of the mean heat flux deviation $\overline{\Delta q}/q$ on the time step $\Delta\tau/\tau_*$ (τ_* is the time of the nonstationary section of the curve q = f(τ)): 1) R = 5 mm, 2) 2.

$$N = (c_0 + c_1 M)\rho M'/2(\lambda_0 + \lambda_1 M), \tag{11}$$

$$G = (4t_0 - 4M - 2NR^2)/R^3, \tag{12}$$

$$S = (3M + NR^2 - 3t_0)/R^4, \tag{13}$$

$$N' = [(2N + 6GR + 12SR^2)(\lambda_0 + \lambda_1 M + \lambda_1 NR^2 + \lambda_1 GR^3 + \lambda_1 SR^4) + (2NR + 3GR^2 + 4SR^3)(2\lambda_1 NR + 3\lambda_1 GR^2 + 4\lambda_1 SR^3)] \times \{\rho R^2 [C_0 + C_1(M + NR^2 + GR^3 + SR^4)]\}^{-1} - \frac{M'}{R^2}. \tag{14}$$

By knowing the time-dependence of the temperature in the section x = 0, the heat flux on the plate surface x = R can be computed by means of (8)-(14). The derivative of the temperature with respect to the time can be found from the formulas:

for the first point

$$M_0' = \frac{1}{2\Delta\tau}(-3t_0 + 4t_1 - t_2), \tag{15}$$

for succeeding points

$$M_k' = (t_{k+1} - t_{k-1})/2\Delta\tau. \tag{16}$$

For constant properties and q = const, the derivative dt/dτ is independent of x for Fo > 5 and equals a const [9]. From (11) we have N' = 0 and the heat flux formula (8) reduces to an expression for the exponential method of determining the constant heat flux [10]

$$q = \rho c R M'. \tag{17}$$

The thermophysical properties of the sensor material can be taken constant for small changes in the temperature of the ordinarily utilized copper calorimeter. In this case c_1 and λ_1 equal zero in (8), (11)-(14).

Confirmation of the proposed method in an example of constant heat flux and material properties showed that the deviation of the computed heat flux from the initial did not exceed 0.14% for q = 3 kW/cm² and Δτ = 0.04 sec. The temperature for x = 0 was found from formula (14) in the paper [9].

The method was used in combination with the exponential method to process the results of heat flux measurement for the sensor inserted in a plasma jet. The sensor was a 3 mm diameter cylinder 5 mm long, heat insulated by a textolite sleeve on the lateral surface side. In conformity with the exponential method a KhK thermocouple was calked into the endface of the calorimetric element and the readings are presented in Fig. 1.

The exponential method yields the value q_c = 2.3 kW/cm^2 on the linear section of the temperature curve. The new method permits measuring the heat flux during the whole sensor heating process (Fig. 1). The heat flux values in the quasistationary heating phase obtained by the two methods are in agreement (discrepancy less than 0.4%). As it turned out, the heat flux about 0.1 sec after sensor insertion varied approximately four times.

The time step utilized can influence the restorable heat flux if incorrectly chosen. As the step diminishes and the number of points grows on the nonstationary section of the curve the heat flux fluctuations increase (Fig. 1). The time step can be selected as a compromise between the number of steps necessary on the nonstationary section and the error of the result (Fig. 2). For an optimal step the error is about 1% in this case.

Computations showed that the optimal step can be shifted substantially towards small values by using a simple method (when the error is still on the sloping section of the curve in Fig. 2). To do this the heat flux values are calculated in terms of the necessary minimal step while the derivatives (15) and (16) are computed using a coarser step within the limits of the smooth section of the temperature curve. The spread in the data is here diminished significantly for other conditions being equal (see Fig. 1).

To confirm the accuracy of restoration of the nonstationary heat flux by the proposed method, the problem (1)-(6) was solved in which condition (3) was replaced by a given function

$$\begin{aligned} &q = 5\tau, \quad 0 < \tau < 0{,}05; \\ &q = 0{,}5 - 5\tau, \quad 0{,}05 < \tau < 0{,}1, \end{aligned} \tag{18}$$

that corresponds to heat flux fluctuations encountered in practice. Equation (1) is replaced by the difference equation

$$\theta_{i,k+1} = \theta_{i,k}\left[1 - \frac{2\left(1+\frac{\lambda_1}{\lambda_0}\theta_{i,k}\right)}{1+\frac{c_1}{c_0}\theta_{i,k}}\Delta\,\mathrm{Fo}\right] + \frac{\left(1+\frac{\lambda_1}{\lambda_0}\theta_{i,k}\right)}{1+\frac{c_1}{c_0}\theta_{i,k}}\Delta\,\mathrm{Fo}\left(\theta_{i+1,k} + \theta_{i-1,k}\right) + \frac{\lambda_1/\lambda_0}{1+\frac{c_1}{c_0}\theta_{i,k}}\frac{\Delta\,\mathrm{Fo}}{4}\left(\theta_{i+1,k} - \theta_{i-1,k}\right)^2, \tag{19}$$

where $\theta = t - t_0$; $\Delta\,\mathrm{Fo} = a_0\Delta\tau/\Delta x^2$.

The boundary conditions were all converted correspondingly. In particular, the temperature on the surface x = R was determined by the dependence

$$\theta_{n1,k} = \theta_{2,k} + \frac{(q_0 + q' k\Delta\tau)\,\Delta x}{\lambda_0 + \lambda_1\theta_{2,k}}, \tag{20}$$

where the coefficients q_0 and q' are found from conditions (18). The temperature field obtained by a numerical solution of the direct problem by using an explicit scheme was utilized to restore the heat flux (Fig. 3). Taking account of the break in the curve at the upper point the accuracy of restoring the heat flux can be considered satisfactory.

The results of restoring the heat flux were compared with data obtained by using two other methods. The small calorimeter method

$$q(\tau) = \delta(c_0 + c_1\theta)\,\rho\,\frac{dt}{d\tau} \tag{21}$$

was the first used, which goes over into the exponential method (17) for constant heat flux. The second was the method of S. Lopata and Ya. Taler [3]. Both yielded satisfactory but

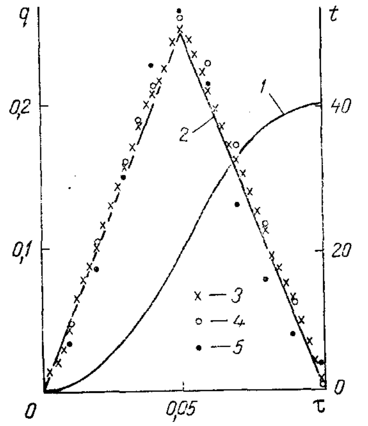

Fig. 3. Time dependence of the temperature and heat flux: 1) temperature in the section x = 0; 2) initial heat flux; 3-5) restored heat flux: 3) by the proposed method; 4) by the small calorimeter method; 5) by the method of S. Lopata and Ya. Taler [3].

somewhat worse results as compared with that assumed (Fig. 3). The temperature on both plate surfaces must be known for the method in [3].

Real values of the temperature utilized to restore the heat fluxes were measured with large and small error, consequently, estimation of the accuracy in determining the heat flux as a function of the temperature error is important. To do this, two and three places were retained in the temperature values computed with numerical model problem examined above.

The heat fluxes restored according to the temperatures known with 2 and 3 place accuracy differed slightly from the heat fluxes computed according to the temperature field calculated in the direct problem with 5 place accuracy. The error in q due to the reason mentioned did not exceed 1-2% in the domain of the peak.

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